A QUESTION OF AUDIENCE, A MATTER OF ADDRESS

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This paper draws attention to certain linguistic features present in notes posted to a common computer data base (Knowledge Forum) by grade four, five and six students in two schools working on a set of mathematical generalising tasks. The notes exhibit some elements present in various sorts of student mathematical writing familiar from other contexts (for instance, accounts of classroom-based mathematical problem solving). One particular feature that this setting might plausibly seem to accentuate is that of the addressivity of the writing, namely the 'turning toward' the other, a feature that is singularly absent from more formal mathematical prose. Nevertheless, despite the relatively young age of these students, their writing also exhibits instances of more 'sophisticated', apparently unaddressed writing.

One of the more taxing questions implicated in the complex interrelationship between language and mathematics has to do with the shaping of form by content and of content by form. One of the less considered aspects of this mutual influence has to do with the nature and influence of the *audience* for the language, especially written mathematical language where the empirical reader (one possible but by no means exclusive audience) may not be co-present with the author, either temporally or spatially. Yet as Bakhtin (1952/1986) was insistent in claiming, every human utterance is addressed *to* someone, a phenomenon he termed *addressivity*, namely an orientation toward the other.

> An essential (constitutive) marker of the utterance is its quality of being directed to someone, its *addressivity*. As distinct from the signifying units of a language – words and sentences – that are impersonal, belonging to nobody and addressed to nobody, the utterance has both an author (and, consequently, expression as we have already discussed) and an addressee. This addressee can be an immediate participant-interlocutor in an everyday dialogue, a differentiated collective of specialists in some particular area of cultural communication, a more or less differentiated public, ethnic group, contemporaries, like-minded people, opponents and enemies, a subordinate, a superior, someone who is lower, higher, familiar, foreign, and so forth. And it can be an indefinite, unconcretized *other* […] (p. 95)

One such question of audience signalled by our paper's title, then, concerns the addressivity of a mathematical text, which is not always a straightforward matter, as John Fauvel (1988) has observed:

> Euclid's attitude [towards the reader] is perfectly straightforward: there is no sign that he notices the existence of readers at all. […] The reader is never addressed. (p. 25)

There are overly-common presumptions about whom a student writer is writing for: for the teacher, for the examiner, for her- or himself, for posterity, for Bakhtin's indefinite other, … . Umberto Eco (1979) has written insightfully about the *model*

author and *model* reader in relation to pedagogic texts (or at least ones with an arguably pedagogic function among others), contrasting it with the empirical reader, say you or me. (For more on this notion in a mathematics education context, see Love and Pimm, 1996.) While these notions are certainly useful in analyzing adult-authored texts intended for students of various ages, it is less clear that they apply to neophyte texts produced by young children, whose awareness of some of the sophistications of and conventions concerning authorship may at best be described as emergent.

1. SOME CONTEXTUAL DETAIL

In this paper, we would like to explore some aspects of mathematical audience by means of written data obtained from grade 4, 5 and 6 students in two Canadian schools, whose classrooms were connected by means of a software environment called *Knowledge Forum* (Scardamalia, 2004; Scardamalia et al., 1994; Moss and Beatty, in press a and b). [1] This paper reports on part of a considerably larger investigation, among other things comparing spoken, face-to-face discussion of mathematical problems with these written counterparts. Due to space, or rather its absence, we focus here only on the written texts.

Knowledge Forum is a networked, multimedia community knowledge space created by community members, in this case *unsupervised and unmediated by the teachers or others*. By authoring *notes*, participants contribute ideas (theories, conjectures, patterns, working models, claims, evidence, data, and so forth) to *views*, which are workspaces for clusters of related activity carried out by the classroom community. Students can either contribute their own notes or co-author them, and have the means to respond to or build onto one another's ideas. Knowledge Forum also has customisable '*scaffolds*'. Examples include: "My theory", "I need to understand", "New information", "This theory cannot explain", "A better theory" and "Putting our knowledge together". These theory-building scaffolds are intended to encourage participants to enter, improve and search community accounts of ideas (Scardamalia, 2004). Activity in the database (reading, writing, building on, etc) is recorded automatically.

Figure 1 on the next page presents one of the views from this study, whose notes were generated in response to the Perimeter Problem by students from two connected grade-four classrooms. The small squares represent student notes and the connecting lines represent *build-ons* to notes created as students read and respond to each other's contributions, thus providing a network expressing connectivity which also codes relative chronology within a conversational thread.

The data-base views are continuously evolving interactive discourse spaces, where each thread of conversation is documented, webs of interchanges graphically displayed, and collective understandings captured as they progress. In this study, two different data bases were used: one for grade 4 students and one for students in grades 5/6. In each data base, six different views were created, one per problem.

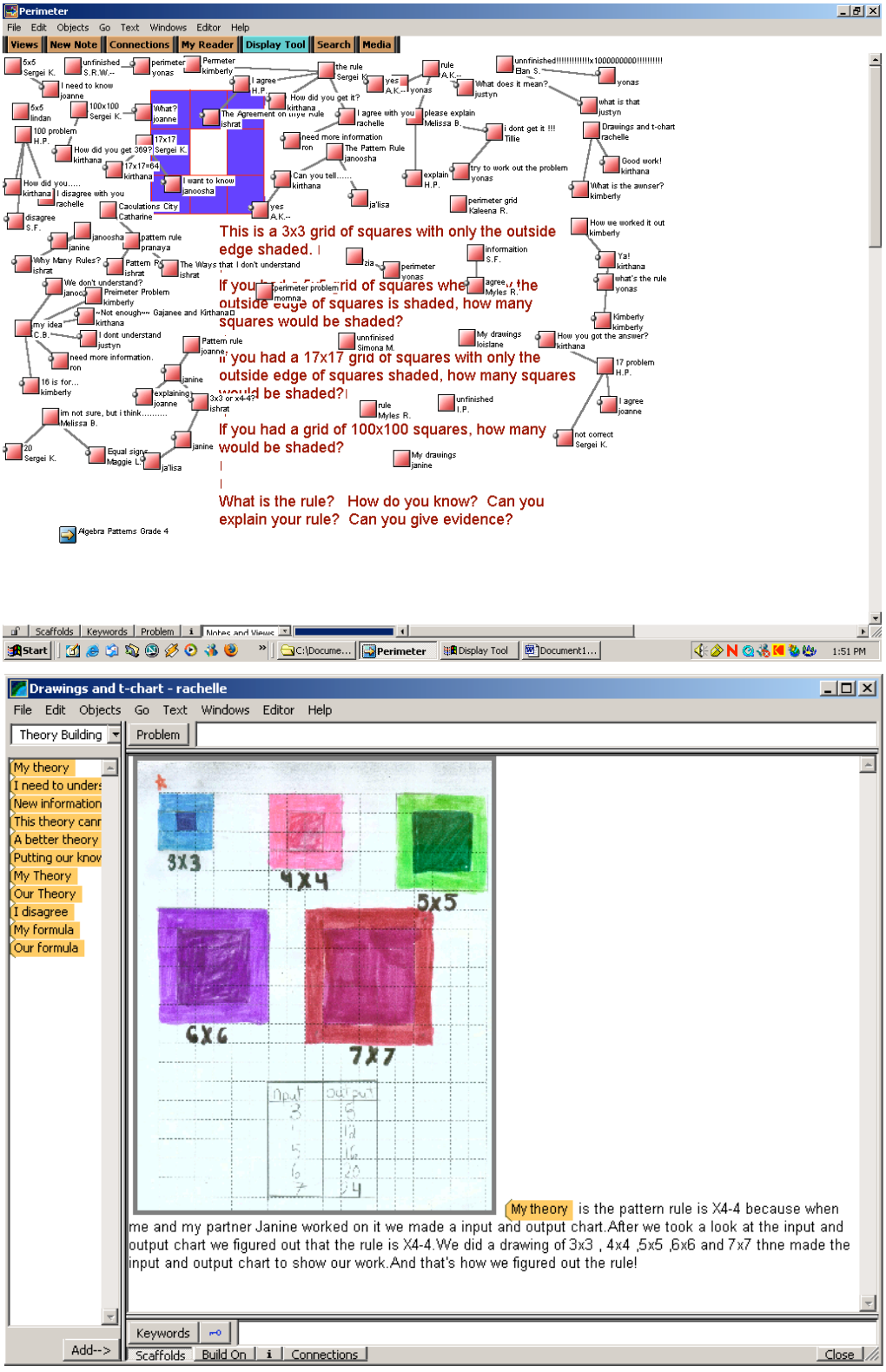

Figure 1: Problem View and a Note

Each note contains a space for composing text (or graphics) and a list of meta-cognitive scaffolds. The sample note in Figure 1 was authored by a student who was contributing a solution to the Perimeter Problem. In this particular note, she used the scaffold *My Theory*.

Students can also use Knowledge Forum's graphics palette to create illustrations or they can scan in drawings, function tables or photographs to support their explanations. These visual representations have two purposes. First, they serve as tools for problem solving; second, they provide students with the means to illustrate and elaborate on their theories. The data base provides a permanent record of each student's thinking; because the discussion is asynchronous, students can revisit their own conjectures or the theories of others at any time.

The 142 participating grade 4, 5 and 6 students came from two schools, an inner-city public school (grade 4, grade 6 classes) ranked as the third most 'needy' by the school district due to a high ESL, low SES population (School A) and a laboratory school (with a grade 4 and a 5/6 class) – School B. The students from the two schools did not know each other and came from different backgrounds both from the perspective of demographics and mathematics instruction, the former school being more traditional and the latter more reform oriented. Prior to working on Knowledge Forum, all students engaged in an extended series of classroom experiences with geometric growth patterns and their expression as part of the intervention study. The generalising problems posted on Knowledge Forum presented students with patterns of growth in different contexts. For each one, students were asked to find the underlying structure and express it as an explicit function or "rule".

The problems that the students worked on were different for the two grades and included linear and quadratic rules embedded in different contexts. All were chosen for this study as a means of developing the student's reasoning about functional relationships.

2. LOOKING AT NOTES THROUGH THEIR DISCOURSE FEATURES

Various discourse features have been identified in terms of their salience for mathematical text, in particular pronouns, deictic markers, hedges, genre elements, verb tense, forms of politeness and modal elements (for a broad survey of this area, see Pimm, 2006; see also Morgan, 1998). One of the complexities of this sort of work is that many of these features are not independent of each other; they co-occur and frequently interact. As a small instance, it has become a commonplace observation that a student expression changing from 'I did' to 'you do' can signal a shift from a narrated account of particular, personal temporally and spatially located experience to an attempt to express a more generalised observation (a linguistic reflection of the triple processes of decontextualisation, depersonalisation and detemporalisation to which Balacheff (1988) draws attention). This instance involves the interaction of a tense change with a pronoun switch. Similarly, variation in modality can be compounded with issues of politeness and hedging.

Returning to the particular data from this study, we start with some general observations about the two classroom pairings. In general, the individual notes from the grade 5/6 classes were more extensive. However, initially at least, the

connectivity in the views remained intra-classroom. The grade fours however, went for shorter cross-classroom notes right away. There was also a noticeable difference between initial postings (which we term *originals*) and any subsequent responses (which we call *build-ons*). Arguably, the author of an initial posting is more aware of writing to the whole community as audience for the note, some of whom they know and others whom they know they do not. “Everyone is going to read this” may be uppermost in the mind or the writer of an original. The subsequent build-on notes in a loop often have greater specificity of address (whether explicitly marked, e.g. “I disagree with you Jessica” or not), even though they are responding to an individual posted message publicly. (This is in contrast with Phillips and Crespo’s 1996 work on grade fours writing a sequence of pen-pal letters to university pre-service elementary teachers, where apart from the teacher-researcher there was no known public audience other than the addressee.)

Many original notes have a certain formality and show an intention of attempting to be very clear and straightforward in presenting an account (and are usually accompanied by some kind of rationalisation, explicitly signalled by the word ‘because’). They are centripetal, that is focused in towards the author and what is to be expressed and need to be centred and self-contained. They contain fewer indeterminate pronouns or other extra-textual elements. By contrast, the response notes are generally more informal and tend to be more specific in focus and in various ways reach out or back to previous authors and their notes.

There were also a number of explicitly hedged elements (see, for instance Rowland, 2000) in the notes, where the author marked his or her uncertainty about or tentativeness of commitment to a claim or assertion being written (the problem involves trapezoidal tables and the number of people who can sit at a growing line of them). As might be expected, they occurred most in originals. The example below (written in response to a problem that asks for the number of people who can sit at any number of trapezoidal tables placed end to end) starts “I’m not sure” (which is not a system support) and repeats three times ‘I think’ something is the case. Nevertheless, the generalisers ‘every time’ and ‘each time’ are used, suggesting a mathematical awareness of the problem’s intent.

> **Add on** – ES (B)
>
> I'm not sure but i think that every time there is another table it adds on 3 because if there is 1table and it=5 and then when it is 2 tables and it=8 and for 3 tables it=11 and for 4 tables it=14
> and for 5 tables it=17.
> So...I think each time they add on 3!
> So I think the rule is:+3

Each note has the scope for a title and an author, as illustrated in our first example above as well as in the note in Figure 1, which is labelled ‘Drawing and t-chart’ and authored by Rachelle (a grade 4 student from School A). If we look at the language of

the sample note in Figure 2, there are a number of discourse features evident. In keeping with the single name in the author line, the prompt ‘My theory’ seemingly individualises the note’s voice to that of a single author (‘Our theory’ was an alternative support-prompt choice), but very quickly we find reference to a second textual presence ‘my partner Janine’, though we do not know whether or not she was physically present when this particular note was being written. Nevertheless, attribution of work reported in evidence for the offered rule was to the pair ‘how we figured out the rule’.

In keeping with conventional mathematical style (see, for instance, Solomon and O’Neill, 1998), the mathematical claim about the particular rule is made in the present tense (as is the claim of this being ‘my theory’), while everything that follows the ‘because’ is in a narratively-structured simple past-tense account, what Marks and Mousley (1990), following Martin, refer to as the ‘report’ genre. [2] The chronology within the account is a little confused (e.g. the sequences marked by ‘when’, ‘after’ and ‘then’ does not seem to indicate a uniform time line). In terms of its content, it is unclear from what was presented whether this is an inductive generalisation based upon the systematic exploration of five consecutive cases (what Balacheff, 1988, would characterise as ‘naïve empiricism) or whether one of these instances could be talked through as a ‘generic example’. This note was posted with the intention of presenting a rule and a rationale, and was neither written in response to another nor followed up upon by others.

There is no apparent explicit addressivity: there is no salutation at the beginning (as, for example, there would be with a ‘friendly letter’ a common genre taught in grade 4), nor apart from the mention of her partner Janine are there any deictic pointers to others (e.g. specific readers, the Knowledge Forum community, …).

The Rule – Uri (School B)

the rule is x3-3. you times the sides and - the corners beacause the corners are not shared within the sides.

Who is right? – Missy (School B)

But whats his name said that it was times 4 plus 1! So whitch one is right? Yours or whats his name?

Other rules – SF (School A)

i agree but you know their could be other rule's like their was in the last view

yes – AK (B)

I agree cus I found 2 rules that work so far.

Different rule – Uri (School B)

there is another rule but explains x3-3 in a diferent way.
i think its actualy number + (n-1) (number-1)+ (n-2)

make sense – Finn (School B)

I agree with you

In this exchange, the expression ‘Whats his name’ suggests a certain address (as the referent indicated is part of the community), but is used between two members of the same class about someone who is not. Missy seems to be assuming everyone has read all the contributions, so the referent will be clear. That said, the referent for the final ‘you’ being agreed with is unclear. This is because although there is a temporal sequence of utterances, there are none of the paralinguistic features of a face-to-face conversation (such as gaze direction or gesture) to assist disambiguation.

In the next linked pair of notes (between two grade 6 students from the same classroom on a different problem), direct address is in evidence, as is a certain pronoun turbulence.

How to figure out this problem – Krishnendu (School A)

My theory is you could multiple column by row togive you an answer but you can dived 2 since the staircase is in half.

Disagreement – Chograb (School A)

I disagree with your theory Krishnendu. IT make a lot of sense but for the 4th position the number of blocks in rows = 4 and number of block in columns is = 4. So 4x4=16 and when you divide that by 2 you get eight. But we need to get 10. I think you are on the right track.

The first note involves a non-standard switch between the modals ‘could’ and can’, but the use of ‘you’ is in the general personalised equivalent of ‘one’. In the second note, there is an interesting double deictic use of the pronoun ‘you’: Krishnendu is directly addressed and the specific reference of ‘your’ in ‘your theory’ is subsequently made clear. The first ‘you’ in when ‘you divide that [16] by 2 you get eight’ we claim is the general ‘anyone’ use common in mathematics. Then the switch to ‘we’ in ‘But we need to get 10’, where there is a common project being (this both is and isn’t Krishnendu’s problem) and then back to very personal specifics of direct address, where Chograb (‘I’) makes a teacher-like comment to Krishnendu (‘you’) in ‘I think you are on the right track’.

Phillips (2001), in her extensive study of grade-four mathematical writing in a variety of non-standard settings, found her students frequently making such encouraging ‘teacherly asides’ as part of the emotional tenor of the directly-addressed feedback on some tasks (e.g. following a set of child-created instructions to make a specific, pre-determined shape from pattern blocks), as if this were a required or desirable feature of classroom writing. It could also be seen as a form of politeness (as, arguably, can ‘IT makes a lot of sense’), an attempt to mitigate somewhat the force of the on-the-record, face-threatening act of explicit disagreement announced both in the note title and the opening words. (For more on Levinson’s account of politeness invoked in a mathematics education context, see Bills, 2000.) Although ‘I disagree’ is a

Knowledge Forum scaffold, students rarely used it explicitly as such, despite frequent overt disagreements.

Our last example illustrates another welter of short responses to an initial posting (the problem concerns the number of faces showing when a rod of *n* cubes is created).

Smiley stickers problem – Mike (A)

My theory is that you would need 5 stickers for 1 cube so for 2 cubes you would have 10 stickers. But for 3 cubes you would have 15 stickers.

How many stickers? – Frank (B)

Can you explain your note better I don't really get what you are saying.

Smile stickers – Jenn (A)

I diagree because a rode of one cube is equal to 6 stickers and the second one was right.

Stickers – Givegga (A)

I disagree with you because a cube has 6 faces so you should have 6 stickers.

6 sides for 1 – Ella (B)

I don't agree because if there was one cube then there would be 6 stickers because one cube has 6 sides because on on the bottum one on the top and four on the sides

(if you don't get what I'm saying look at this okay, also I'm drawing the cubes sides making the cube ok):

do not count – Shauna (B)

there are 6 stickers on 1 cube but when you put another cube it makes 10 because the stickers that are beside eachother do not count

six sided cube – SRW (A)

My theory is that on one cube there would be 6 sickers eecause there are 6 sides.

1 cube six sides – JI (A)

With 1 cube there would be 6 sides. With 2 cubes there would be 10 sides. With 3 cubes here would be 14 sides. And on and on.

14 stickers – Finn(B)

I disagreee, for 3 cubes you would need 14 stickers

A full analysis of these exchanges is beyond the scope of this paper, but the pronouns alone make for an interesting challenge in keeping track of reference, to say nothing of the claims, counter-claims and reasons offered in response to the first post. And on and on.

IN SUMMARY

By means of exploring a range of student responses generated in a particular, computer-mediated communicative setting (Knowledge Forum), we have been able to highlight instances of addressivity between the texts, where an author leans towards her audience in more or less explicit and tacit ways. However, Bakhtin's claim about addressivity being a core element of any communication raises the question of why in more formal mathematical prose such elements are systematically absent. We are left wondering to whom such a denatured mathematical utterance-text is addressed, stripped of pronouns and time and other specific ties to the deictic world of the human here-and-now. Students working in this environment reveal some inclination toward explicit addressivity of their notes about their mathematical claims, theories and findings. But they also, already in grade 4, show a certain attunedness to more formal mathematical conventions by which mathematical utterances are both marked and expressed. And finally, are we all fated, as mathematical readers, to remain, in the words of the poet Robert Kroetsch (1989), "eavesdroppers on an address to the sacred" (p. 163)?

NOTES

[1] The image of the Roman *forum* is interesting, conjuring a place where people may come together freely to discuss matters of public or common import. It is also the origin of the term *forensic*, the sort of examination we intend to subject the student data to, in order to present in the public setting of CERME.

[2] This also fits well Bruner's (1986) dichotomy between *narrative* and *paradigmatic* styles.

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